# Collective Oscillations in Superconducting Thin Films in the Presence of Vortices

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## Abstract

A plasma wave propagates inside an anisotropic superconducting film sandwiched between two semi-infinite non-conducting bounding dieletric media. Along the c-axis, perpendicular to the film surfaces, an external magnetic field is applied. We show how vortices, known to cause dissipation and change the penetration depth, affect the propagative mode. We obtain the complex wave number of this mode and, using $YBCO$ at 4 $K$ as an example, determine a region where vortex contribution is dominant and dissipation is small.

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## I. INTRODUCTION

In the plasma frequency, a collective oscillation of the electron gas in the positive ionic background occurs, which is fundamental to understand the electromagnetic properties of conductors. Below the plasma frequency the conductor reflects the incident electromagnetic radiation and, above it becomes transparent thus allowing a propagative mode. For metals the plasma frequency is typically found in the ultraviolet region ($10^{15} - 10^{16}\ Hz$)[1].

A well-known feature of superconductors is the existence of a gap, the energy required to break a Cooper pair in the ground state condensate. Typically the frequency associated to the gap for conventional superconductors is in the $10^{11}\ Hz$ range, whereas for the new high-Tc superconductors is one order magnitude higher[2].

The question whether the superconductor can support collective modes *without* inducing pair breaking effects is an old one and has been discussed since the early days of the theory of superconductivity[3,4]. Apart from the so-called Carlson-Goldman mode, which happens under special circumstances[5,6], any other attempt to excite collective modes in isotropic *bulk* superconductors leads to the destruction of the superconducting state. This follows the well-known argument[4] that the Coulomb interaction shifts the frequency of such oscillations, the plasma frequency, to above the gap frequency. However, it was recently shown that highly anisotropic superconductors do display a plasma oscillation below the superconducting gap[7]. This oscillation, specially to the layered structure, is due to the Josephson coupling between the superconducting planes.

Plasma modes in superconductors, isotropic or not, has been recently revisited from another point of view. Plasma modes below the gap are possible without destroying the superconducting state, as long as, they propagate in the interface between the superconductor and a non-conducting bounding medium of very high dieletric constant. This is the so-called superficial plasma mode[8,9], which is made possible by the charges located at the interface of the superconductor and the dieletric medium, responsible for the creation of an electric field concentrated mainly outside the superconductor.

In a thin film, the coupling between the two superficial plasma modes yields two possible branches, a symmetric and a anti-symmetric, similarly to metals[15−17] and semiconductors[18]. The film thickness must be smaller than the London penetration depth in order to produce this coupling. Oscillations between the kinetic energy of the superelectrons and the electrical field energy take place in these modes and for this reason they are called *plasma modes*. The lower frequency branch was predicted for superconductors[10,12,11] some time ago and observed in thin granular aluminium films, in the hundreds of $MHz$ range[13], and in thin $YBa_2Cu_3O_{7-x}$ films[14], in the higher frequency range of hundreds of $GHz$. The highest frequency branch is predicted to be within experimental observation range for the high Tc materials[19]. In case of highly anisotropic superconducting materials, measurements of such upper and lower branches are expected to give information on the transverse and the longitudinal London penetration depths, respectively[19]. In conclusion, plasma modes in superconducting films can be an important tool for the probe of many intrinsic properties of superconductors.

Long ago Gittleman and Rosenblum[20] have studied the effects of an applied external current at the radio and microwave frequency range into pinned vortices and obtained the surface impedance. For an $AC$ applied magnetic field and in the weak pinning regime, Campbell[21] showed that the effect of vortices can be described by a new $AC$ London penetration depth, whose square is the original London penetration depth squared plus a new term describing the elastic interaction of vortices with the pinning centers. A few years ago these models were generalized to convey the effects of creep[22] and to provide a more detailed description of the elastic properties of the vortex lattice near a surface[23].

In this paper we study the effects of a constant uniform magnetic field, applied perpendicularly to the thin film, into the thin film propagative mode. A sufficiently large magnetic field allows the thermodynamic stability of a vortex system, which influences the collective oscillations, affecting considerably the above modes. The vortices are induced into an oscillatory dissipative motion around their pinning centers. This motion couples to the electromagnetic fields resulting either into an underdamped or an overdamped regime. This paper

is developed in the context of independent vortex and superelectron degrees of freedom. We understand by superelectron current, any supercurrent other than that one necessary to bring the thermodynamic equilibrium of vortices. The vortex degree of freedom, described by its position in space, also represents its intrinsic current. In this framework arises the question whether the superelectron or the vortex contribution dominates the propagative mode behavior. Hereafter, by plasma mode we refer to the limit where superelectron contribution is the largest. So, pure plasma modes are found in the complete absence of an applied magnetic field. In this paper we discuss conditions that render the modes underdamped and vortex dominated. This is the case of interest because the attenuated oscillations can be regarded as taking place between the vortex pinning energy and the electrical field energy.

The present work is done in the simplest possible theoretical framework, essentially a generalization of the Gittleman-Rosenblum[24,25], such that vortices and superelectrons are independently coupled to Maxwell's theory. Here we are mainly interested in the low temperature regime and therefore, ignore the contribution of normal electrons to the problem. Thus, wave damping is only due to the vortex dissipative motion.

We consider here an anisotropic superconductor with its uniaxial direction (c-axis) orthogonal to the film surfaces: the two London penetration depths, transverse ($\lambda_\perp$) and longitudinal ($\lambda_\parallel$) to the surfaces give an anisotropy such that $\lambda_\perp/\lambda_\parallel > 1$ . There are two dielectric constants, the non-conducting medium and the superconductor ones, $\tilde{\varepsilon}$ and $\varepsilon_s$, respectively. Thus we are assigning to the superconductor a frequency independent dielectric constant. We refer to the speed of light in the dielectric as $v = c/\sqrt{\tilde{\varepsilon}}$. The uniform static applied magnetic field is $H_0$. For each individual vortex, the viscous drag coefficient is $\eta_0$ and the elastic restoring force constant (Labusch parameter) is $\alpha_0$. Their ratio, $\omega_0 \equiv \alpha_0/\eta_0$, is the so-called depinning frequency, above which dissipation becomes dominant in the vortex motion. To have coupling between the two surfaces the film thickness, $d$, must be smaller than $\lambda_\parallel$.

The choice of a nonconducting bounding medium of very high dieletric constant is crucial to lower the frequency range of the modes to below the gap frequency. For this reason we

take $SrTiO_3$ as the bounding media, whose dieletric constant is known to be high up to the $GHz$ frequency[13] at low temperatures: $\tilde{\varepsilon} \approx 2.0\ 10^4$. Then the speed of light in the dielectric, $v = 2.1\,10^6\ ms^{-1}$, is substantially smaller than $c$. Our work is restricted to identical top and bottom dielectric media, which does not imply lack of generality. Similar conclusions should also apply to the general asymmetric case.

This paper is organized as follows. In the next section(II) we introduce the major equations describing the film mode in the presence of vortices. Its dispersion relation is analytically derived under some justifiable approximation. In section III we apply our model to the high-Tc superconductor $YBa_2Cu_3O_{7-\delta}$, investigating a range of parameters such that the lowest energy film mode is mostly associated to the vortex dynamics, but yet remains underdamped. Finally, in section IV, we summarize our major results.

## II. PROPAGATING MODES IN SUPERCONDUCTING FILMS WITH PERPENDICULAR MAGNETIC FIELD

In this section we introduce the basic equations governing wave propagation in a superconducting film sandwiched between two identical non-conducting dielectric media and subjected to an uniform static magnetic field perpendicularly applied to the film surface. An external electromagnetic wave of angular frequency $\omega$ and vacuum wavenumber $k \equiv \omega/c$ is inserted in the dielectric bounded film. We determine the dispersion relation of the lowest energy film mode, whose imaginary part reveals the attenuation behavior. Phenomenological theories, such as the present one, only describe the superconductor in a energy range much lower than the pair breaking threshold.

The electromagnetic dynamics of fields and superelectrons is described by the Maxwell's equations,

$$\nabla \cdot\ \vec{D} = e\,(n_s - \bar{n}_s) \tag{1}$$

$$\nabla \cdot\ \vec{H} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \tag{3}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{4}$$

$$\tag{5}$$

and consequently by the continuity equation,

$$\nabla \cdot \vec{J} + e\,\frac{\partial n_s}{\partial t} = 0 \tag{6}$$

where $n_s$ represents the space and time dependent charge density, $\bar{n}_s$ is its equilibrium value and $e$ stands for the electron charge. The distinction between $n_s$ and $\bar{n}_s$ is necessary because, propagation through the system disrupts the neutrality, as seen from Gauss' law $(n_s - \bar{n}_s = 0)$, and the local charge density is no longer constant.

As previously noticed the contribution of vortices and of superelectrons are independent in the present model. The field $\vec{J}$, the superelectron current density involved in net macroscopic transport, and the field $\vec{u}$, the vortex displacement from its equilibrium position are independent in the present model. Hence the supercurrent density $\vec{J}$ corresponds to a macroscopic average of the total superelectron motion, where the supercurrent necessary to establish each vortex averages to zero. This approximation, valid for the present purposes, cannot give any information on the supercurrent distribution surrounding each vortex line.

The simplest possible model that treats the response of the vortices to the presence of a supercurrent external to them is the harmonic approximation of Gittleman and Rosenblum[20],

$$\eta_0 \frac{\partial \vec{u}}{\partial t} + \alpha_0 \vec{u} = \Phi_0 (\vec{J} \times \hat{n}) \tag{7}$$

where $\hat{n}$ is a unit vector parallel to the flux lines mean direction. ¿From its turn, the displacements of vortices from their equilibrium positions affect the propagating electromagnetic wave. Fiory and Hebard[24] have considered this question and found that besides the kinetic inductance due to the superelectrons, the moving vortices also contribute, producing an electric field inside the superconductor.

$$\vec{E} = \mu_0 \lambda^2 \cdot \frac{\partial \vec{J}}{\partial t} - \mu_0 H_0 (\frac{\partial \vec{u}}{\partial t} \times \hat{n}) \tag{8}$$

The assumption of anisotropy yields a tensorial London penetration depth.

$$\lambda = \begin{pmatrix} \lambda_\perp & 0 & 0 \\ 0 & \lambda_\parallel & 0 \\ 0 & 0 & \lambda_\parallel \end{pmatrix} \quad \lambda_\perp = \sqrt{\frac{m_\perp}{\mu_0 \bar{n}_s e^2}} \quad \lambda_\parallel = \sqrt{\frac{m_\parallel}{\mu_0 \bar{n}_s e^2}} \tag{9}$$

We pick a coordinate system where the two plane parallel surfaces separating the superconducting film from the dielectric medium are at $x = d/2$ and $x = -d/2$, such that $\hat{n} \equiv \hat{x}$ and propagation is along the $z$ axis. Vortex displacement is described by a vector field parallel to the surfaces, $\vec{u} = u_y \hat{y} + u_z \hat{z}$, with no orthogonal components to them ($u_x = 0$). According to symmetry arguments, all fields for the present geometry can be expressed as $F_i(x)\ \exp\left[-\imath(q\, z - \omega\, t)\right]$, where the wave number $q$ have yet to be determined. Because vortex motion is dissipative, the wave's amplitude decays exponentially with distance, and one obtains for the fields's expression $F_i(x)\ \exp\left(q''\, z\right)\ \exp\left[-\imath(q'\, z - \omega\, t)\right]$. Then wave number is a complex number, $q = q' + \imath q''$.

Solving Maxwell's equations for the chosen geometry gives two independent sets of field components, the transverse electric (TE) and the transverse magnetic (TM) propagating modes. In the former the non-zero electromagnetic field components are $H_x$, $E_y$ and $H_z$, the non-vanishing supercurrent is $J_y$ and the vortex displacement is along the direction of wave propagation ($u_z$). This is an extremely high frequency mode in the present theory, and so not interesting because it lies above the gap. For the latter, the non-zero electromagnetic field components are $E_x$, $H_y$ and $E_z$, the non-vanishing supercurrent components are $J_x$ and $J_z$ and the propagating wave displaces the vortices perpendicularly to its direction of propagation ($u_y$). This is a very interesting mode because it supports low frequency propagating waves. The major difference between TE and TM modes is that the latter displays superficial charge densities at the film-dielectric interfaces and the former does not. Such superficial charge densities stem from the supercurrent component orthogonal to the film surface, $J_x$, which is discontinuous at the interfaces, thus rendering a strong coupling between the superconducting film and the bounding media.

Introducing the time dependence $\exp\left(\imath\, \omega\, t\right)$ into Eq.(7) and Eq.(8) results in a change

of the penetration depth parallel to the surfaces due to the vortex contribution[23,21].

$$\imath\,\omega\,\mu_0\lambda_\perp^2\,J_x = E_x, \qquad \imath\,\omega\,\mu_0\bar{\lambda}_\parallel^2\,J_z = E_z \tag{10}$$

$$\bar{\lambda}_\parallel^2 = \lambda_\parallel^2 + (\frac{B_0\,\Phi_0}{\mu_0\,\alpha_0})\frac{1}{1+\imath(\omega/\omega_0)} \tag{11}$$

This equation shows that vortices and superelectrons contribute additively to the parallel penetration depth. Notice the depinning frequency $\omega_0$ establishes two distinct physical regions for the vortices response. For $\omega \ll \omega_0$ dissipation is weak and $\bar{\lambda}_\parallel$ is essentially a real number. For $\omega \geq \omega_0$ and a sufficiently large magnetic field, dissipation dominates the vortices response because $\bar{\lambda}_\parallel$ is complex.

The superconductor's dielectric constant is tensorial, $\vec{D} = \epsilon_0\varepsilon_s\vec{E} - \imath\vec{J}/\omega = \epsilon_0\varepsilon\cdot\vec{E}$ and for the TM mode we have that

$$\varepsilon_x = \varepsilon_s - \frac{1}{(k\lambda_\perp)^2}, \quad \varepsilon_z = \varepsilon_s - \frac{1}{(k\bar{\lambda}_\parallel)^2}, \quad k \equiv \frac{\omega}{c} \tag{12}$$

The TM field equations for the dielectric medium, ($x \geq d/2$ and $x \leq -d/2$), are given bellow:

$$E_x = \imath\frac{q}{\tilde{\tau}^2}\,\frac{\partial E_z}{\partial x}, \quad H_y = \imath\,\epsilon_0\,\frac{\omega\,\tilde{\varepsilon}}{\tilde{\tau}^2}\,\frac{\partial E_z}{\partial x}, \quad \frac{\partial^2 E_z}{\partial x^2} - \tilde{\tau}^2\,E_z = 0, \quad \tilde{\tau}^2 = q^2 - k^2\tilde{\varepsilon} \tag{13}$$

and the ones for the superconducting film ($-d/2 \leq x \leq d/2$) follow.

$$E_x = \imath\frac{q\,\varepsilon_z}{\tau^2\,\varepsilon_x}\,\frac{\partial E_z}{\partial x}, \quad H_y = \imath\,\epsilon_0\,\frac{\omega\,\varepsilon_z}{\tau^2}\,\frac{\partial E_z}{\partial x}, \quad \frac{\partial^2 E_z}{\partial x^2} - \tau^2\,E_z = 0 \quad \tau^2 = \frac{\varepsilon_z}{\varepsilon_x}q^2 - k^2\varepsilon_z \tag{14}$$

The dispersion relations follow from the continuity of the ratio $H_y/E_z$ at a single interface, say $x = d/2$, once assumed the longitudinal field $E_z$ has a definite symmetry. It happens in this way because, the superconductor film is bounded by the same dielectric medium in both sides. Solving Eq.(13) one gets that above the film ($x \geq d/2$),

$$E_z = \tilde{E}_o \exp\left(-\tilde{\tau}\,x\right) \quad and \quad \frac{\tilde{H}_y}{\tilde{E}_z}|_{x=d/2} = -\imath\frac{\omega\epsilon_0\tilde{\varepsilon}}{\tilde{\tau}} \tag{15}$$

¿From Eq.(14) we learn that for the superconducting film ($-d/2 \leq x \leq d/2$) there are two possible states, symmetrical and anti-symmetrical, where the longitudinal field is expressed

by $E_z = E_o \cosh(\tau\, x)$ and $E_z = E_o \sinh(\tau\, x)$, respectively. As discussed earlier, we shall only consider the symmetric branch, the lowest mode in energy. So the ratio of the tangential fields becomes $H_y/E_z|_{x=d/2} = \imath\omega\epsilon_0\tilde{\varepsilon}_z \tanh(\tau d/2)/\tau$. Continuity of this ratio across the interface gives the following implicit relation.

$$\frac{\tau\,\tilde{\varepsilon}}{\tilde{\tau}\,\varepsilon_z} = -\tanh(\tau\,\frac{d}{2}) \tag{16}$$

To find the dispersion relation we must solve Eq.(16). Here we use an approximate method to analytically solve it. This approximation amounts to replace the function $(\tanh z)/z$ in Eq.(16), by another function, $1/\sqrt{1+(2/3)z^2}$, which has an extremely close behavior. For $z \ll 1$ both functions coincide up to the second order term in the Taylor series expansion: $1-(1/3)z^2+\ldots$. As all our results are derived in the range $z \ll 1$ thus, we replace Eq.(16) by the following approximate dispersion relation.

$$\frac{\tilde{\varepsilon}}{\tilde{\tau}} \approx -\frac{d\varepsilon_z}{2}\frac{1}{\sqrt{1+\frac{2}{3}(\frac{\tau d}{2})^2}} \tag{17}$$

Squaring the above expression, one obtains a linear equation for $q^2$:

$$q^2 = (\frac{\omega}{v})^2\frac{1+(\frac{2\omega\bar{\lambda}_\parallel}{dv})^2[\bar{\lambda}_\parallel^2+\frac{d^2}{6}]}{1-\frac{2}{3}(\frac{\omega}{v})^4\bar{\lambda}_\parallel^2\lambda_\perp^2} \tag{18}$$

The term proportional to $d^2/6$ in the numerator is irrelevant, assuming the film much thinner than the penetration depth ($\lambda_\parallel \gg d$). We restrict the present study to frequencies much below the assymptotic frequency ($(\frac{\omega}{v})^4\bar{\lambda}_\parallel^2\lambda_\perp^2 \ll 1$), thus obtaining the following dispersion relation:

$$q^2 = (\frac{\omega}{v})^2(1+(\frac{2\omega\bar{\lambda}_\parallel^2}{dv})^2) \tag{19}$$

In the absence of an applied uniform magnetic field ($H_0 = 0$), consequently with no vortices, there is no dissipation and $q'' = 0$. In this case we retrieve the well-known dispersion relation of plasma modes taking into account the retardation effect[11,13].

Next we study two different behaviors of the dispersion relation in the presence of vortices.

<u>*optical mode*</u> At low frequencies the mode is, in leading order, a plane wave travelling in the dielectric medium, $q' \approx \omega/v$, with no attenuation along the direction of propagation, ($q'' \approx 0$). Perpendicularly to the film, the amplitude shows no attenuation, because $\tilde{\tau} \approx 0$, according to Eq.(15). We obtain, from the Taylor expansion of Eq.(18), the lowest order corrections in $\omega$ to the above description of the optical regime.

$$q' = \frac{\omega}{v}\{1 + \frac{1}{2}[\frac{2\omega(\lambda_\parallel^2 + \frac{B_0\ \Phi_0}{\mu_0\ \alpha_0})}{dv}]^2\} + \cdots \tag{20}$$

$$q'' = -\frac{4\omega^4 \frac{B_0\ \Phi_0}{\mu_0\ \alpha_0}(\lambda_\parallel^2 + \frac{B_0\ \Phi_0}{\mu_0\ \alpha_0})}{v^3 d^2 \omega_0} + \cdots \tag{21}$$

<u>*coupled mode*</u> For sufficiently large frequencies, Eq.(19) no more describes a linear response. In this range the superconducting film and the dieletric media are effectively coupled, which implies in a reduction of the mode propagation speed ($(\omega/q')/v \ll 1$). This is the most interesting regime since film and dielectric produce a low energy mode.

Far-way from the linear regime, and provided that the asymptotic frequency is still out of range, Eq.(19) is approximately described by its second term, resulting into the dispersion relation $q \approx (2/d)[\omega/(v\bar{\lambda}_\parallel)]^2$. ¿From this, we obtain its wavevector and attenuation:

$$q'(\omega) = \frac{2\ \omega^2}{d\ v^2}\ [\lambda_\parallel^2 + \frac{B_0\ \Phi_0}{\mu_0\ \alpha_0}\ \frac{1}{1 + (\omega/\omega_0)^2}] \tag{22}$$

$$q''(\omega) = -\frac{B_0\ \Phi_0}{\mu_0\ \alpha_0}\frac{\omega^3/(\omega_0 v^2 d)}{1 + (\omega/\omega_0)^2} \tag{23}$$

In the frequency range where the above dispersion relation is a valid approximation, the ratio between the real and the imaginary parts of the London penetration depth, determines whether the mode is overdamped or underdamped: $q'/q'' = -Re(\bar{\lambda}_\parallel^2)/Im(\bar{\lambda}_\parallel^2)$.

The cross-over magnetic field,

$$B_1 \equiv \frac{\lambda_\parallel^2\ \ \mu_0\ \alpha_0}{\Phi_0} \tag{24}$$

splits the regimes of superelectron ($B_0 \ll B_1$), and vortex ($B_0 \gg B_1$) dominance. In these limits Eq.(11) can be replaced by approximated expressions, $\bar{\lambda}_\parallel^2 \approx \lambda_\parallel^2$, and $\bar{\lambda}_\parallel^2 \approx (B_0\ \Phi_0/\mu_0\ \alpha_0)/(1 + \imath(\omega/\omega_0))$, respectively. Recall the assumption of the present model that

the superelectron contribution is never dissipative. If in addition to an applied magnetic field much larger than $B_1$, we choose a frequency range $\omega < \omega_0$, then $q'' < q'$ and the mode is underdamped. The dispersion relation follows a square root dependence, and becomes,

$$\omega^2 \approx \frac{dv^2\mu_0\alpha_0}{2B_0\phi_0}q' \tag{25}$$

In this interesting limit the mode energy shows many oscillations between vortex and electric field energies before dissipation dominates. At higher frequency ($\omega > \omega_0$) this is no longer possible, since the mode becomes overdamped due to the large dissipation of vortices above the depinning frequency. The frequency $\omega_0$ coarsely defines a cross-over region between the underdamped and the overdamped regimes.

In the next section, using experimental parameters measured on $YBCO$, we search for favorable conditions in frequency and magnetic field to observe underdamped coupled modes on a thin film.

## III. YBCO THIN FILM

In this section a $YBa_2Cu_3O_{7-\delta}$ thin film is taken, as an example, to determine a frequency and magnetic field window where the mode is coupled, underdamped and vortex dominated. The wave must be underdamped in order to travel over many wavelengths before its amplitude is completely attenuated. For this high-Tc superconductor the anisotropy ($\lambda_\perp/\lambda_\parallel = 5$) and the zero-temperature London penetration depth along the $CuO_2$ planes are well-known.[26,27]. At very low temperature several experiments[28] have determined the viscosity and the Labusch constant, all giving the same numbers, which are summarized in table I. Such parameters have a temperature dependence[29], not taken into account here because we only consider a fixed low temperature, namely, 4 $K$. The magnetic field dependence of the Labusch constant, known to exist for high-Tc materials[30] and low-Tc ones[31], is not considered either. For this discussion we choose the film thickness $d = 10$ $nm$.

Table I : Properties of the high-Tc material $YBa_2Cu_3O_{7-\delta}$ at T = $4\,K$

| $\alpha_0$ $(N/m^2)$ | $\eta_0$ $(N\,s/m^2)$ | $\omega_0 = \frac{\alpha_0}{\eta_0}$ $(10^9 \quad rad/s)$ | $\lambda_{\parallel}$ $(\mu m)$ | $B_1$ $(T)$ |
|---|---|---|---|---|
| 3.0 $10^5$ | 1.2 $10^{-6}$ | 250 | 0.15 | 4.1 |

Fig.1 provides a pictorial intuitive view of the wave propagation inside the superconducting film for the TM symmetric propagating mode. Dimensions are out of proportion in order to enhance some of the most relevant features. Only the electric field lines inside the superconducting film are shown. The superficial charges are also shown and, represents the sources of this propagating electric field. The electric field lines show a very important feature of this wave[19], namely, the supercurrent component along the wave propagation direction, $J_z$, is dominant over $J_x$. A magnetic field perpendicularly applied to the film surfaces produces vortices, pictorially represented at the top surface. The oscillatory displacement suffered by vortices, because of the driving Lorentz force caused by $J_z$ (Eq.(7)), is also shown in this figure.

As previously discussed, the adequate choice of frequency and magnetic field windows is fundamental to observe the lower energy mode. We can distinguish several different regions within the $B_0$ vs. $\omega$ diagram. Fig.2 shows such regions for $YBCO$, according to the above parameters. Two cross-over lines separate this diagram in three different regions: the optical regime, the underdamped coupled regime and the overdamped coupled regime.

The lower line in Fig.2, called $\omega_{cr}$, separates the optical region from the coupled regions. This cross-over line is defined through Eq.(20), using as conditon that the second term becomes a non negligible fraction $\chi_1$ of the first term and so can no longer be ignored,

$$\omega_{cr} = \sqrt{\frac{\chi_1}{2}} \frac{dv}{\lambda_{\parallel}^2 + \frac{B_0\,\Phi_0}{\mu_0\,\alpha_0}} \tag{26}$$

We have arbitrarily chosen ten percent ($\chi_1 = 0.1$) as our criterion for the optical mode boundary.

The upper line in Fig.2, called $\omega_{d\pm}$, is related to the dissipation and separates the underdamped to the overdamped regimes. The criterion for dissipation is the ratio $q'/q''$, which for the coupled regime, is approximately given by the ratio between the real and the imaginary part of the squared penetration depth $\bar{\lambda}_{\|}^2$ (Eq.(11)), according to Eq.(23). Thus our second cross-over line is defined by $Im(\bar{\lambda}_{\|}^2) = \chi_2 Re(\bar{\lambda}_{\|}^2)$ where $\chi_2$ is an arbitrary factor. This condition gives a second degree equation for $\omega/\omega_0$, $\chi_2\lambda_{\|}^2(\omega/\omega_0)^2 - (B_0\Phi_0/\alpha_0\mu_0)(\omega/\omega_0) + \chi_2(\lambda_{\|}^2 + B_0\Phi_0/\alpha_0\mu_0) = 0$, whose solutions, $\omega_{d\pm}(B_0)$, form the upper and lower branches of a single curve that encircles the overdamped regime area.

$$\frac{\omega_{d\pm}}{\omega_0} = \frac{B_0}{2\chi_2 B_1} \pm \sqrt{(\frac{B_0}{2\chi_2 B_1})^2 - \frac{B_0}{B_1} - 1} \tag{27}$$

Therefore, the dissipative region demands a minimum applied field $B_2$ to exist, defined by the vanishing of the above square root:

$$B_2 = 2\chi_2(\chi_2 + \sqrt{1+\chi_2^2})B_1 \tag{28}$$

Hence the two curves $\omega_+$ and $\omega_-$ have a common start at $(B_2, \omega_2)$, where $\omega_2 = (\chi_2 + \sqrt{1+\chi_2^2})\omega_0$, and approach the asymptotic lines $(\omega_0/\chi_2)(B_0/B_1)$ and $\omega_0\chi_2$, respectively. For the diagram in Fig.2, we have taken $\chi_2 = 0.5$ thus, obtaining that $B_2 \approx 6.64\ T$ and $\omega_2 \approx 4,05\ 10^{11}\ rad/s$. The asymptotic lines become $\omega_{d+}/\omega_0 \to 2(B_0/B_1)$ and $\omega_{d-}/\omega_0 \to 0.5$.

As indicated by the $B_0$ vs. $\omega$ Fig.2 diagram, the modes are optical for frequencies below the $\omega_{cr}$ line where they are weakly affected by the superconductor properties and the vortex dynamics. In this region and for $B_0 \ll B_1$ the superelectron dominates over the vortex response and, effectively, there are plasma modes. For $B_0 \gg B_1$, the $\omega_{cr}$ line decreases inversely proportional to $B_0$. Above the $\omega_{d-}$ line, and, at large magnetic fields, $B > B_2$, the modes become overdamped. Thus the interesting region lies above the $\omega_{cr}$ line and below the $\omega_{d-}$ line, where the modes are underdamped coupled and vortex dominated. In this intermediate region, dissipation should be small enough ($q' > q''$) to allow wave propagation over some wavelengths before attenuation sets in.

All Figures discussed below were obtained using Eq.(19) expression. The complex wave number $q$ is then easily derived as a function of $\omega$.

Fig.3 shows the dispersion relation $\omega$ vs. $q'$ for $B_0 = 0T$ and $B_0 = 20T$. In case of zero magnetic field, The frequency window considered in this figure is below the zero magnetic field optical-coupled crossover ($\omega_{cr} \approx 2,10\ 10^{11}\ rad/s$). Indeed, the $B_0 = 0T$ mode shows a quasi linear dependence. However for a magnetic field $B_0 = 20T$ , the presence of vortices changes dramatically the dispersion relation. The frequency $\omega_{cr}$ has droped substantially, according to this figure. Below the mode is optical, similarly to the zero magnetic field case, and above the mode is slow in comparison to the zero field one. This effect clearly comes from the vortex overwhelming contribution at this large magnetic field value. In this frequency window, the mode is underdamped until the frequency $\omega_{d-}$ is reached. Above it turns to be overdamped. In order to better estimate the attenuation, we have plotted the ratio $q'/q''$ for the same frequency window (Fig.4). Notice that $q'/q''$, obtained from Eq.(19) and shown here, gives directly the mode attenuation, whereas Eq.(23) just provides an approximate criterion, used to define the dissipative curve $\omega_{d\pm}$ of Fig.2. Fig.4 shows that for $\omega_{cr} < \omega < \omega_{d-}$ the mode propagates over various wavelengths before its amplitude goes to zero. According to Fig.4 $q'/q''$ diverges for low frequencies within the optical regime. This behavior is explained recalling that all losses are caused by vortices and disappear at zero frequency.

The reduced speed, defined as the ratio between the phase velocity and the speed of light in the dielectric, $(\omega/q')/v$ is plotted in Fig.5. In the optical regime this ratio is essentially equal to one. This is quite verifiable at zero magnetic field but not at $20T$, where the modes are strongly slowered by the presence of vortices.

## IV. CONCLUSION

In this paper we have studied superficial coupled modes in a superconducting film surrounded by two identical dielectric media with an applied magnetic field perpendicular to the surface. The superconductor is anisotropic and its uniaxial direction (c-axis) is perpendicular to the interfaces with the dielectric medium. The choice of non-conducting media

of high dielectric constant helps to lower the propagating wave frequency range much below the gap frequency. We consider a static magnetic field above the lowest critical field, that allows for the existence of pinned and dissipative vortices. In the present approach superelectrons and vortices contribute additively to the impedance. Vortices and superelectrons interact with each other through the Lorentz force and through an electric field, created by vortex motion and superelectrons acceleration. Here we have studied how the lowest energy branch, the TM symmetric mode, is affected by vortices. Under a justifiable approximation, we obtain an analytical expression for this dispersion relation, which can describe simultaneously the three different possible behaviors for a propagating mode in a superconducting film subjected to an exterior magnetic field, namely, optical regime, underdamped coupled regime and overdamped coupled regime.

We find that in very high magnetic field, vortices dominate over the superelectrons response. The modes are well described by the vortex oscillations around their pinning centers where, their energy oscillates between the pinning energy and the electrical one. We have studied the $B_0$ vs. $\omega$ diagram for a very thin superconducting film, made of the high-Tc material $YBCO$. We find three different regions: optical, underdamped coupled and overdamped modes. Nested between the optical and the overdamped regions, and above a certain critical magnetic field cross-over, is the region of interest. There exist, in this frequency and magnetic field window, underdamped propagative modes, whose behavior is determined by the vortex response, and not by the superelectrons.

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# FIGURES

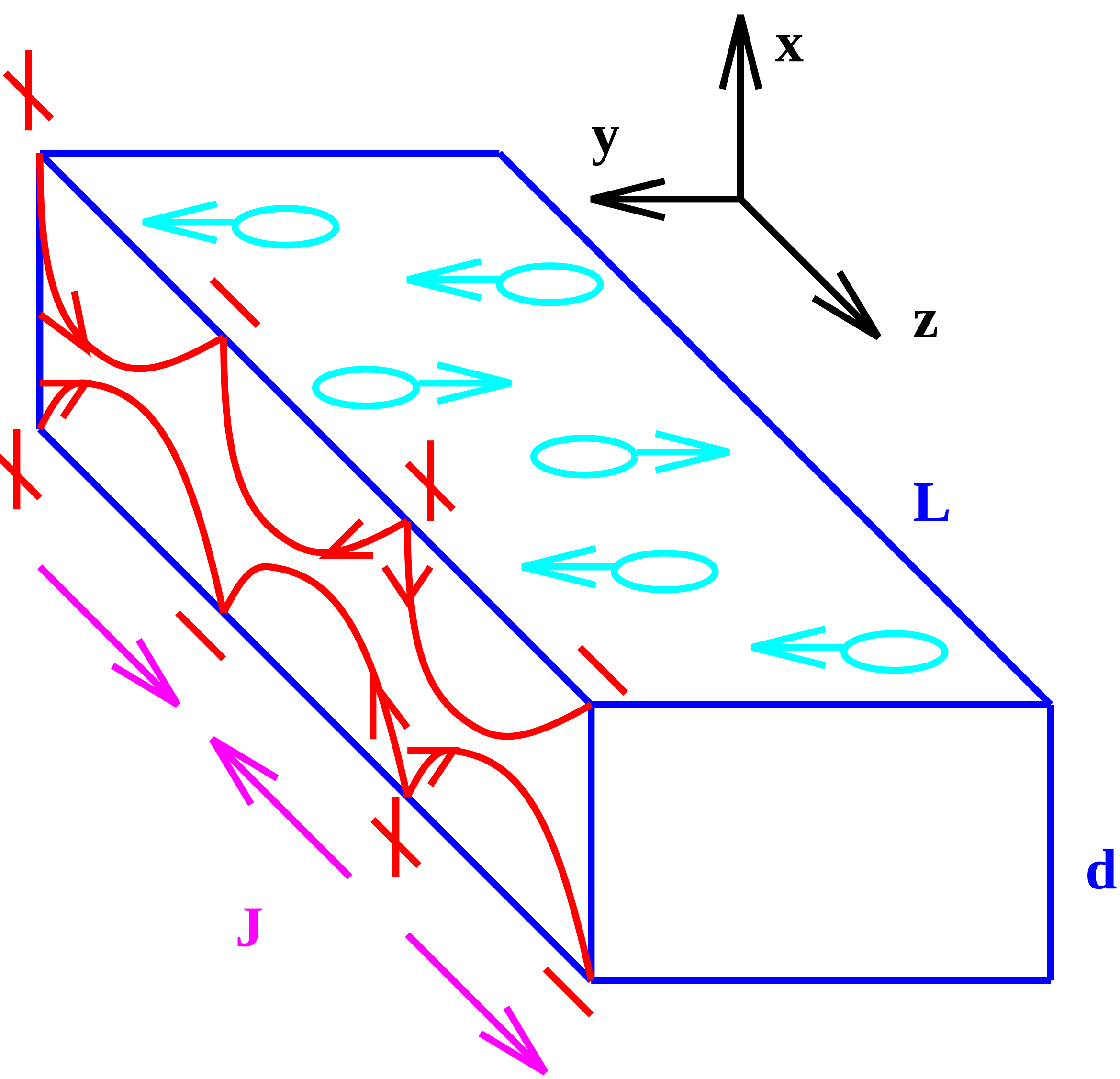
FIG. 1. A pictorial view of wave propagation in a superconducting film surrounded by identical non-conducting media in both sides. Scales are out of proportion in order to enhance some of the features. The instantaneous electric field is shown here only inside the film. The superficial charge densities and the motion of the vortex lines are also sketched here.

FIG. 2. The diagram $B$ vs. $\omega$ for a very thin $YBCO$ superconducting film, $d = 10nm$-thick, surrounded by the dielectric material $SrTiO_3$ shows three regions: optical, underdamped coupled and overdamped modes. The dashed line separates the superelectron (below) to the vortex (above) dominated regime.

FIG. 3. Dispersion relation $\omega$ versus $q'$ for a 10 $nm$ $YBCO$ film. In this frequency range and for zero magnetic field the dispersion relation is purely optical. For $B_0 = 20\ T$ , the modes are associated to the vortex dynamic and are underdamped until the frequency $\omega_d$ is reached.

FIG. 4. The ratio $q'/q''$ is displayed here versus $\omega$ showing the mode damping for the same frequency range of Fig.3. The ratio, although undergoes a dramatic change in this range, is always larger than one, thus signaling underdamped behavior.

FIG. 5. The retardation ratio, $(\omega/q')/v$, is shown for the frequency range of Fig.4. For zero applied field this ratio is near one showing that mode is essentially optical. This is not case for $B_0 = 20\ T$ whose strong deviation from one signals coupling between the dielectric and the superconducting film due to the presence of vortices.


x
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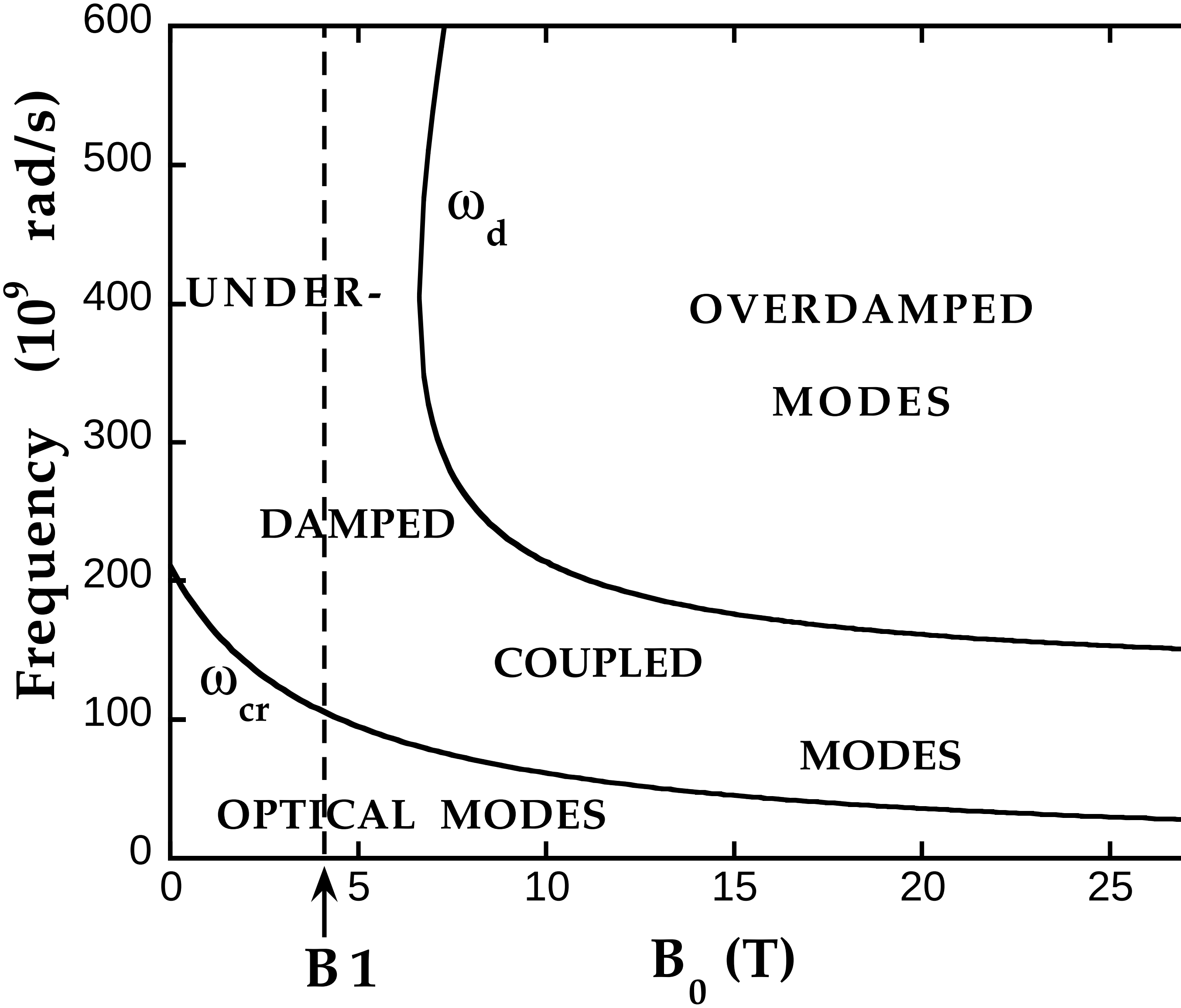

600
500
400
300
200
100
0
Frequency (10^9 rad/s)
0
5
10
15
20
25
B 1
B0 (T)
ωd
ωcr
UNDER-
DAMPED
OPTICAL MODES
OVERDAMPED
MODES
COUPLED
MODES

Fig.3 Collective Oscillation...

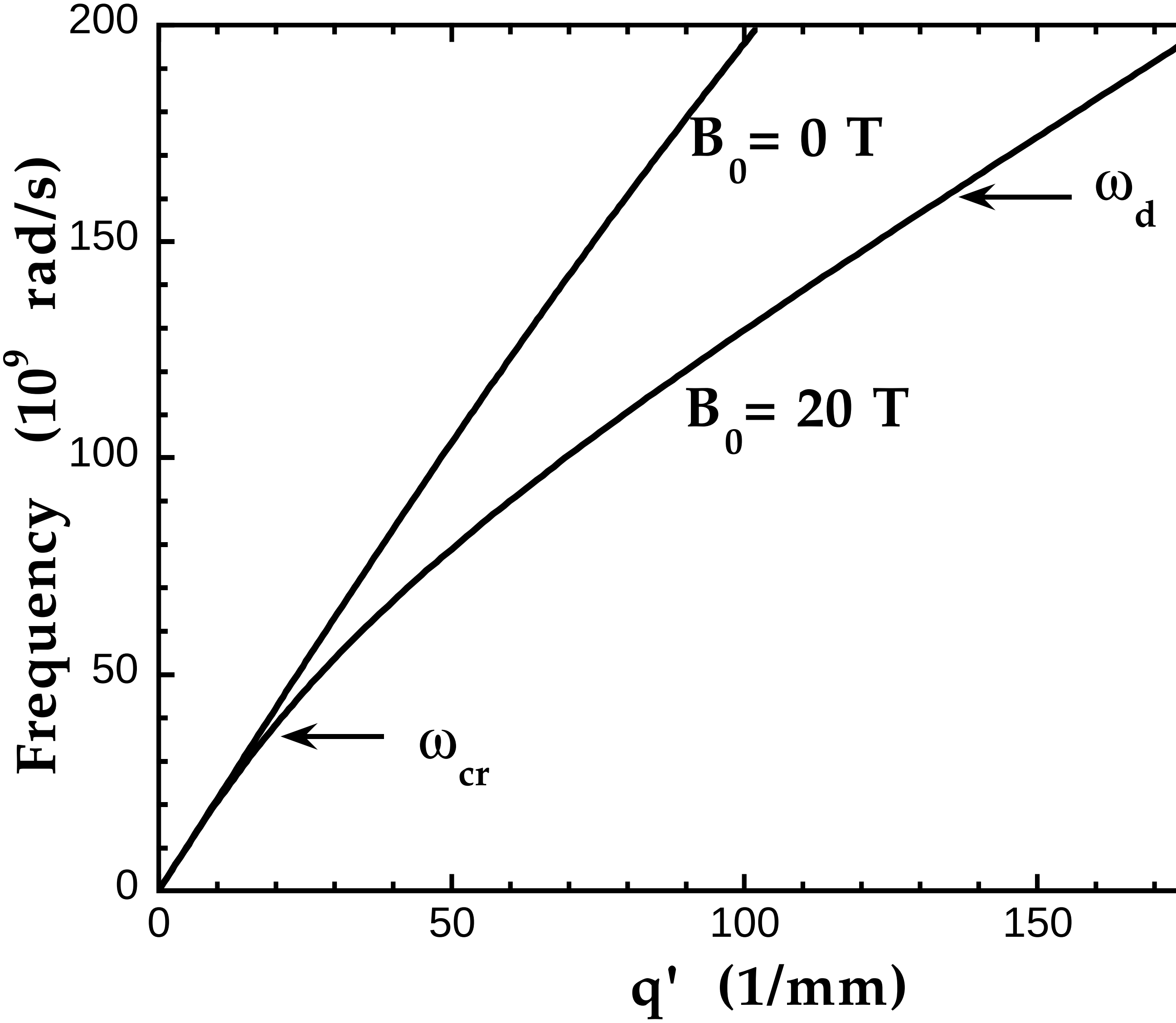

Fig.4 Collective Oscillation...

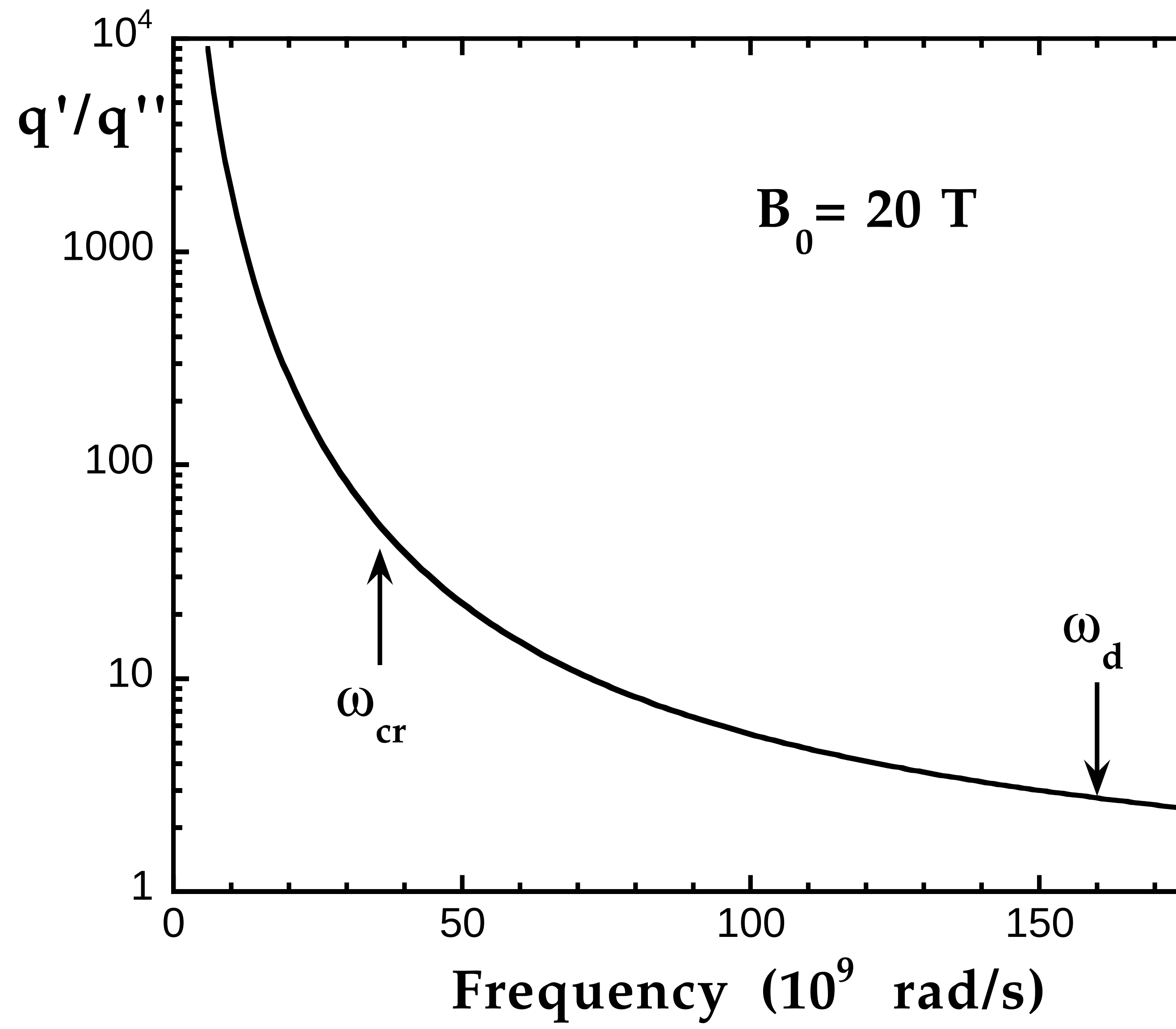

Fig.5 Collective Oscillation...